

(Following Paper ID and Roll No. to be filled in your Answer Books)

Paper ID: 199419

Roll No. 

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**B. TECH.****Theory Examination (Semester-IV) 2015-16****ENGG MATHEMATICS-III****Time : 3 Hours****Max. Marks : 100****Section-A**

1. **Attempt all questions of this section. Each question carry equal marks.** (2×10 = 20)
- (a) Write the cauchy's Reimaun conditions in polar coordinates system.
- (b) Write the statement of generalized cauchy's integral formula for  $n^{\text{th}}$  derivative of an analytic function at the point  $Z = Z_0$ .
- (c) Find the  $Z$  - transform of  $U_n = \{a^n\}$
- (d) Write the normal equations to fit a curve  $y = ax^2 + b$  by least square method.

(e) If covariance between  $x$  and  $y$  variable is 10 and the variance of  $x$  and  $y$  are respectively 16 and 9, find the coefficient of correlation.

(f) The regression equations calculated from a given set of observations for two random variable are

$$x = -0.4y + 6.4 \text{ and } y = -0.6x + 4.6 \text{ calculate mean values of } x \text{ and } y.$$

(g) Write the Newton's Raphson iterative formula to find the value of  $\sqrt{N}$ .

(h) Find the missing data in the given table :

$x$	0	1	2	3
$f(x)$	580	556	-	465

(i) If  $f(n)$  is given in following table :

$x$	0	0.5	1
$f(x)$	1	0.8	0.5

then using trapezoidal rule, evaluate

$$\int f(x) dx$$

(j) Find the third forward difference with the arguments 2, 4, 6, 8 of the function  $f(x) = x^3 - 2x$

(2)

### Section-B

2. Attempt any five questions from this section.

(10×5 = 50)

(a) Find the Laurent series for the function

$$f(z) = \frac{7z^2 + 9z - 18}{z^3 - 9z}, Z \text{ is complex variable}$$

valid for the regions

(i)  $0 < |z| < 3$

(ii)  $|z| > 3$

(b) Using calculus of residue, evaluate the following integral

$$\int_0^{\infty} \frac{dx}{(a^2 + x^2)^2}$$

(c) Find the inverse Fourier sine transform of  $\frac{1}{x} e^{-ax}$

(d) Using least square method, fit a second degree polynomial from the following data :

$x$	0	1	2	3	4	5	6	7	8
$y$	12.0	10.5	10.0	8.0	7.0	8.0	7.5	8.5	9.0

Also estimate  $y$  at  $x = 6.5$

(3)

- (e) For the following data, calculate the finite differences and obtain the forward and backward difference polynomials. Also interpolate at  $x = 0.25$  and  $x = 0.35$

$x$	0.1	0.2	0.3	0.4	0.5
$f(x)$	1.40	1.56	1.76	2.00	2.28

- (f) Construct the divided difference table for the data.

$x$	0.5	1.5	3.0	5.0	6.5	8.0
$f(x)$	1.62	5.87	31.0	131.0	282.12	521.0

Hence find the interpolating polynomial and an approximation to the value of  $f(z)$ .

- (g) Solve the system of equations  $AX=B$ , where

$$A = \begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} -10 \\ 8 \\ 7 \\ -5 \end{bmatrix}$$

using the LU decomposition method. Take all the diagonal elements of L as 1.

(4)

- (h) Solve the initial value problem

$$\frac{dy}{dx} = -2xy^2, y(0) = 1$$

with  $h = 0.1$  on the interval  $[0, 0.3]$ . Use the fourth order Runge-Kutta method.

### Section-C

**Note: Attempt any two questions from this section. Each question carry equal marks. (15×2=30)**

3. (a) Show that for the function give as -

$$f(z) = \begin{cases} \frac{2xy(x+iy)}{x^2+y^2} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$$

The C-R conditions are satisfied at origin but derivative of  $f(z)$  at origin does not exist.

- (b) Verify that the function on  $4(xy) = xy$  is harmonic and find its conjugate harmonic function. Express  $u+iv$  as an analytic function  $f(z)$ .

$$u = x^2 - y^2 - y$$

(5)

- (c) Find the Fourier transform of Block function  $f(t)$  of height 1 and duration  $a$  defined by

$$f(t) = \begin{cases} 1 & \text{for } |t| \leq \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

4. (a) Using Z - transform, solve the difference equation

$$u_{n+2} - 4u_{n+1} + 3u_n = 5^n$$

with  $u_0 = u_1 = 1$

- (b) The first four moments of a distribution about  $x = 4$  are 1, 4, 10, 45. Comment on the skewness and Kurtosis of the distribution.

- (c) For 10 observations on price ( $x$ ) and supply ( $y$ ) the following data were obtained

$$\Sigma x = 130, \Sigma y = 220, \Sigma x^2 = 2288$$

$$\Sigma x^2 = 5506 \text{ and } \Sigma_{xy} = 3467$$

Obtain the two lines of regression.

5. (a) Find the root of the equation  $xe^x = 3$  by regula falsi method correct up to two decimal places in the interval (1, 1.5).

- (b) Prove the following identities :

(i)  $\left(\frac{\Delta^2}{E}\right)\mu_x \neq \frac{\Delta^2\mu_x}{E\mu_x}$

(ii)  $\left(\frac{\Delta^2}{E}\right)e^x \cdot \frac{E(e^x)}{\Delta^2 e^x} = e^x$

- (c) The velocity  $v$  of a particle at distance  $s$  from a point on its path is given by the following table :

$s$ (m.)	0	10	20	30	40	50	60
$v$ (m./s.)	47	58	64	65	61	52	38

Estimate the time taken to travel 60m. Using Simpson's one-third rule.