

**B. TECH**  
**(SEM-III) THEORY EXAMINATION 2019-20**  
**MATHEMATICS-IV**

Time: 3 Hours

Total Marks: 100

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

## SECTION A

1. Attempt all questions in brief.

2 x 10 = 20

Q no.	Question	Marks	CO								
a.	Solve the following partial differential equation $yq - xp = z$ .	2	1								
b.	Solve the Cauchy's problem $u_x - u_y = 0$ . $u(x, 0) = x$	2	1								
c.	Classify the following equation. $x^2 \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = u$	2	2								
d.	Solve the partial differential equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} = 0$ .	2	2								
e.	Find the median of 6, 8, 9, 10, 11, 12, 13.	2	3								
f.	The first three central moments of a distribution are 0, 15, -31. Find the moment of coefficient of skewness.	2	3								
g.	If the p.m.f of a discrete random variable X is <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr> <td style="padding: 5px;">X</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">3</td> </tr> <tr> <td style="padding: 5px;">f(x)</td> <td style="padding: 5px;"><math>\frac{1}{2}</math></td> <td style="padding: 5px;"><math>\frac{1}{3}</math></td> <td style="padding: 5px;"><math>\frac{1}{6}</math></td> </tr> </table> Determine E(X) and V(X).	X	1	2	3	f(x)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$	2	4
X	1	2	3								
f(x)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$								
h.	The probability density function f(x) of a continuous random variable X is defined by $f(x) = \begin{cases} \frac{A}{x^2}, & 5 \leq x \leq 10 \\ 0, & \text{otherwise} \end{cases}$ Find the value of A.	2	4								
i.	Find the mean of the Binomial Distribution $B\left(4, \frac{1}{3}\right)$ .	2	4								
j.	A machine which produces mica insulating washers for use in electric device to turn out washers having a thickness of 10 mm. A sample of 10 washers has an average thickness 9.52 mm with a standard deviation of 0.6 mm. Find out t.	2	5								

## SECTION B

2. Attempt any three of the following:

3 x 10 = 30

Q no.	Question	Marks	CO
a.	Solve $(D^2 - DD' - 2D'^2)z = (y-1)e^x$	10	1
b.	A rectangular plate with insulated surface is 10 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature along the short edge $y=0$ is given by: $u(x, 0) = \begin{cases} 20x & 0 \leq x \leq 5 \\ 20(10-x) & 5 < x < 10 \end{cases}$ While the two edges $x=0$ and $x=10$ as well as the other short edge are kept at $0^\circ\text{C}$ . Find the steady state temperature at any point (x,y) of the plate.	10	2

c.	Find an exponential curve $PV^k = k$ for the data:	10	3																								
	<table border="1" style="margin: auto; border-collapse: collapse;"> <tr><td>V</td><td>50</td><td>100</td><td>150</td><td>200</td></tr> <tr><td>P</td><td>135</td><td>48</td><td>26</td><td>17</td></tr> </table>	V	50	100	150	200	P	135	48	26	17																
V	50	100	150	200																							
P	135	48	26	17																							
d.	Fit a Poisson distribution to the following data which give the number of yeast cells per square for 400 squares	10	4																								
	<table border="1" style="margin: auto; border-collapse: collapse;"> <tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>F</td><td>103</td><td>143</td><td>98</td><td>42</td><td>8</td><td>4</td><td>2</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> </table> <p>It is given that <math>e^{-1.52} = 0.2674</math>.</p>	X	0	1	2	3	4	5	6	7	8	9	10	F	103	143	98	42	8	4	2	0	0	0	0		
X	0	1	2	3	4	5	6	7	8	9	10																
F	103	143	98	42	8	4	2	0	0	0	0																
e.	To test the effectiveness of inoculation against cholera, the following table was obtained	10	5																								
	<table border="1" style="margin: auto; border-collapse: collapse;"> <tr><td></td><td>Attached</td><td>Not attached</td><td>Total</td></tr> <tr><td>Inoculated</td><td>30</td><td>160</td><td>190</td></tr> <tr><td>Not inoculated</td><td>140</td><td>460</td><td>600</td></tr> <tr><td>Total</td><td>170</td><td>620</td><td>790</td></tr> </table> <p>(The figure represents the number of persons) Use Chi square test to defend or refute the statement. The inoculation prevents attack from cholera. The value of <math>\chi^2</math> for 1 degree of freedom at 5% level is 3.841.</p>		Attached	Not attached	Total	Inoculated	30	160	190	Not inoculated	140	460	600	Total	170	620	790										
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3. Attempt any **one** part of the following: 1 x 10 = 10

Q no.	Question	Marks	CO
a.	Solve $(D + 1)(D + D' - 1)z = \sin(2x + 3y)$	10	1
b.	In a partial destroyed laboratory record of an analysis of correlation data, the following result only are legible: Variance of $x = 9$ Regression equation: $8x - 10y + 66 = 0$ , $40x - 18y = 214$ . What were (a) the mean value of $x$ and $y$ (b) the standard deviation of $y$ and the co-efficient of correlation between $x$ and $y$ ?	10	3

4. Attempt any **one** part of the following: 1 x 10 = 10

Q no.	Question	Marks	CO
a.	Solve $x^2 \frac{\partial^2 z}{\partial x^2} - 4y^2 \frac{\partial^2 z}{\partial y^2} - 4y \frac{\partial z}{\partial y} - z = x^2 y^2 \log y$	10	1
b.	A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y = y_0 \sin^3 \frac{\pi x}{l}$ . If it is released from rest from this position, find the displacement $y(x,t)$ .	10	2

5. Attempt any **one** part of the following: 1 x 10 = 10

Q no.	Question	Marks	CO
a.	An insulated rod of length $l$ is ends A and B maintained at $0^\circ\text{C}$ and $100^\circ\text{C}$ respectively until the steady state condition prevails. If B is suddenly reduced to $0^\circ\text{C}$ and maintained at $0^\circ\text{C}$ , Find the temperature at a distance $x$ from A at time $t$ .	10	2

b.	Find the multiple regression equation of $X_1$ on $X_2$ and $X_3$ from the data Given below:	10	3																					
	<table border="1" style="margin: auto; border-collapse: collapse;"> <tr><td><math>X_1</math></td><td>3</td><td>5</td><td>6</td><td>8</td><td>12</td><td>10</td></tr> <tr><td><math>X_2</math></td><td>10</td><td>10</td><td>5</td><td>7</td><td>5</td><td>2</td></tr> <tr><td><math>X_3</math></td><td>20</td><td>25</td><td>15</td><td>16</td><td>15</td><td>2</td></tr> </table>	$X_1$	3	5	6	8	12	10	$X_2$	10	10	5	7	5	2	$X_3$	20	25	15	16	15	2		
$X_1$	3	5	6	8	12	10																		
$X_2$	10	10	5	7	5	2																		
$X_3$	20	25	15	16	15	2																		

6. Attempt any one part of the following: 1 x 10 = 10

Q no.	Question	Marks	CO
a.	State the Bayes' theorem. The probability that a civilian can hit a target is $\frac{2}{5}$ and the probability that an army officer can hit the same target is $\frac{3}{5}$ . While the civilian can fire 8 shots in the time, the army officer fires 10 shots. If they fire together, then what is the probability that army officer shoots the target?	10	4
b.	Define the Normal distribution. The daily wages of 1000 workers are distributed around a mean of Rs. 140 and with a standard deviation of Rs. 10. Estimate the number of workers whose daily wage will be (i) between Rs. 140 and Rs. 144, (ii) less than Rs. 126 (iii) more than Rs. 160.	10	4

7. Attempt any one part of the following: 1 x 10 = 10

Q no.	Question	Marks	CO																																				
a.	An IT company wants to appoint an effective trainer to improve the performance of their engineers. Four groups of 7, 8, 10 and 11 engineers from total 36 engineers were given 5 days training by the 4 trainers. Scores were awarded to the engineers at the end of the training on their skills. Let us examine the preference of one engineer of one trainer over other three trainers. Given that $\alpha = 0.05$ i.e. at 5% level of significance the value of $F(3,32) = 3.29$ .	10	5																																				
b.	Distinguish between p chart and C chart. The number of defectives in 17 samples of size 500 each from 17 lots is shown below:	10	5																																				
	<table border="1" style="margin: auto; border-collapse: collapse;"> <tr> <td style="text-align: center;">Samples</td> <td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td> </tr> <tr> <td style="text-align: center;">No. of defectives</td> <td>20</td><td>25</td><td>35</td><td>45</td><td>15</td><td>65</td><td>15</td><td>20</td><td>35</td><td>25</td><td>12</td><td>9</td><td>21</td><td>22</td><td>32</td><td>35</td><td>38</td> </tr> </table>	Samples	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	No. of defectives	20	25	35	45	15	65	15	20	35	25	12	9	21	22	32	35	38		
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No. of defectives	20	25	35	45	15	65	15	20	35	25	12	9	21	22	32	35	38																						
	Find out the control limits for the number of defective units and also check whether the process is under control or not.																																						