

(Following Paper ID and Roll No. to be filled in your Answer Book)

Paper ID : 110302

Roll No.

B.Tech.

(SEM. III) THEORY EXAMINATION. 2015-16

DISCRETE STRUCTURES AND GRAPH THEORY

[Time : 3 hours]

[Total Marks : 100]

Section-A

1. Attempt all parts. All parts carry equal marks. Write answers of each section in short. (10x2=20)
 - (a) Define multiset and power set. Determine the power set $A = \{1, 2\}$.
 - (b) Show that $[(p \vee q) \Rightarrow r] (\sim p) \Rightarrow (q=r)$ is tautology or contradiction.
 - (c) State and prove pigeon hole principle.
 - (d) Show that if set A has 3 elements, then we can have 26 symmetric relation on A.
 - (e) Prove that $(P \vee Q) \rightarrow (P \wedge Q)$ is logically equivalent to $P \leftrightarrow Q$.

- (f) How many 4 digit numbers can be formed by using the digits 2, 4, 6, 8 when repetition of digits is allowed.
- (g) The converse of a statements is: **If a steel rod is stretched, then it has been heated.** Write the inverse of the statement.
- (h) **If a and b are any two elements of group G then prove $(ab)^{-1}=(b^{-1}a^{-1})$.**
- (i) If $f: A \rightarrow B$ is one-one onto mapping, then prove that $f^{-1}: B \rightarrow A$ will be one-one onto mapping.
- (j) Write the following in DNF $(x+y)(x'+y')$.

Section-B

Attempt **any five** questions. (10×5=50)

2. If D_n define the set of all positive odd integers, i.e. $D_n = \{1, 3, 5, \dots\}$, then prove with the help of mathematical induction $P(n) : 1+3n$ is divisible by 4.
3. Solve the recurrence relation using generating function: $a_n - 7a_{n-1} + 10a_{n-2} = 0$ with $a_0 = 3, a_1 = 3$.

4. Express the following statements using quantifiers and logical connectives.
- (a) **Mathematics book that is published in India has a blue cover.**
- (b) All animals are mortal. All human being are animal. Therefore, all human being are mortal.
- (c) There exists a mathematics book with a cover that is not blue.
- (d) He eats crackers only if he drinks milk.
- (e) There are mathematics books that are published outside India.
- (f) **Not all books have bibliographies.**
5. Draw the Haase digram of $[p(a, b, c), \leq]$, (Note: ' \leq ' stands for subset). Find greatest element, least element, minimal element and maximal element.
6. Simplify the following boolean expressions using k map:
- a) $Y = ((AB)' + A' + AB)'$
- b) $A'B'C'D' + A'B'C'D + A'B'CD + A'B'B'CD' = A'B'$

7. Let G be the set of all non-zero real number and let $a*b=ab/2$. Show that $(G,*)$ be an abelian group.
8. The following relation on $A=\{1, 2, 3, 4\}$. Dtermine whether the following :
- a) $R = \{(1,3), (3,1), (1,1), (1,2), (3,3), (4,4)\}$,
- b) $R=AXA$
9. If the permutation of the elements of $\{1,2,3,4,5\}$ are given by $a=(1\ 2\ 3)(4\ 5)$, $b=(1)(2)(3)(4\ 5)$, $c=(1\ 5\ 2\ 4)(3)$. Find the value of x , if $ax=b$. And also prove that the set $Z_4 = \{0,1,2,3\}$ is a commutative ring with respect to the binary modulo operation $+4$ and $*4$.

Section-C

Attempt **any two** questions. (2×15=30)

10. Let L be a bounded distributed lattice, prove if a complement exists, it is unique. Is D_{12} a complemented lattice? Draw the Hasse diagram of $[P(a,b,c), \leq]$, (Note: ' \leq ' stands for subset). Find greatest element, least element, minimal element and maximal element.

11. Determine whether each of these functions is a bijection from R to R .
- (a) $f(x) = x^2 + 1$
- (b) $f(x) = x^3$
- (c) $f(x) = (x^2 + 1)/(x^2 + 2)$
12. a) Prove that inverse of each element in a group is unique.
- b) Show that $G = \langle (1, 2, 4, 5, 7, 8), X^9 \rangle$ is cyclic. How many generators are there? What are they?

—x—